Math 3435: Things to know for the midterm

1. How to determine if a PDE is nonlinear, linear homogeneous, linear inhomogeneous.

2. How linearity of a PDE affects solutions (that you can add solutions to get new solutions, find solution to an inhomogeneous linear PDE as a sum of a particular solution and solutions to the corresponding homogeneous PDE, etc.).

3. How to solve a first order linear PDE by using characteristics. The book (Section 1.2) gives what it calls the “geometric method” and the “coordinate method” – knowing either one is fine. You should be able to do constant and variable coefficient cases, and be able to find a solution given initial data.

4. What well-posed means (existence, uniqueness and stability of solutions). You should also know that equations can be well-posed for a region and ill-posed for another region (eg heat equation is well-posed for $t > 0$ and ill-posed for $t < 0$). You should know that there are different ways to measure stability, but you do not need to know the two ways we measured distance between data and solutions – if you were asked to say anything about stability you would be given information about the distance in the problem.

5. The basic second order equations: heat/diffusion $u_t = ku_{xx}$, wave $u_{tt} = c^2u_{xx}$, and Laplace $u_{xx} = 0$. The fact that time-invariant solutions of heat and wave equations are solutions of the Laplace equation. The fact that derivatives of solutions to these equations are also solutions.

6. The fact that all linear 2nd order PDE are essentially equivalent to one of the three basic equations, which are referred to as parabolic (heat eqn), hyperbolic (wave) and elliptic (Laplace). You do not need to be able to do the change of variables that shows this.

7. For a given 2nd order linear PDE, how to determine whether it is parabolic, hyperbolic or elliptic using the sign of the determinant of the matrix of 2nd order coefficients (or an equivalent formula). You should be able to do this on the whole plane in the constant coefficient case and also be able to find regions of the plane on which different behaviors occur in the variable coefficient case.

8. The general solution formula $u(x,t) = f(x + ct) + g(x - ct)$ for the wave equation, and that (if $c > 0$) the term $f(x+ct)$ represents a wave moving left and $g(x-ct)$ a wave moving right. You should be able to use this equation to solve problems, but you do not need to know how to derive from it the formula for the solution with given initial data.

9. How to use the formula for the solution of the wave equation with given initial data. The formula will be on the formula sheet (last page of this document).

10. The fact that information propagated by the wave equation moves at speed $\leq c$. You need to know how to use this in problems and how it is expressed in the formula for the solution from initial data.

11. How to draw graphs of solutions of the wave equation from initial data.

12. That the wave equation conserves energy in the sense that $E'(t) = 0$ (see formula sheet for the energy $E(t)$), and how to verify this using the equation and integration by parts.

13. How to use energy to show that solutions of the wave equation are unique.

14. What the maximum principle for the heat equation says, and how to use it to show that a solution is bounded from bounds on its initial and boundary data. You do not need to know the proof of the maximum principle.

15. How to use the maximum principle to show that solutions of the heat equation are unique.

16. How to use energy in the heat equation to show that solutions are unique.

17. How to use the solution formula for the heat equation to solve problems when given initial data. How to write solutions in terms of the error function.

18. The following basic features of the heat and wave equations. Wave equation has finite propagation speed, information is transported, it is well-posed for all time, has energy conservation, and singularities move along characteristic lines. Heat equation has infinite propagation speed, information is lost, it is well-posed for $t > 0$ and ill-posed for $t < 0$, energy and solutions decay in time and are instantly smooth (there are no singularities).

19. How to use odd or even reflection to solve the heat equation on $(0, \infty)$ with initial data and Dirichlet or Neumann boundary data. How to write these solutions in terms of the error function.

20. How to use odd or even reflection to solve the wave equation on $(0, \infty)$ with initial data and Dirichlet or Neumann boundary data. How to graph these solutions.